Title: Estimation of Traffic Delays and Vehicle Queues at Freeway Work Zones

Author: Yi Jiang
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by

Yi Jiang
Purdue University
Department of Building Construction Management
1414 Knoy Hall of Technology
West Lafayette, IN 47907-1414

E-mail: yjiang@tech.purdue.edu
Phone: 765-497-3338
FAX: 765-496-2246
ABSTRACT

The traffic flow is disrupted and delayed at a work zone because the traffic capacity and the vehicle speed are lower at the work zone section than at other portions of the roadway. Traffic delays at a work zone include delays caused by deceleration of vehicles while approaching the work zone, reduced vehicle speed through the work zone, time needed for vehicles to resume freeway speed after exiting the work zone, and vehicle queues formed at the work zone. Delay equations were developed in this study for both when arrival traffic flow is above the work zone capacity and below it. In addition to equations of work zone traffic delays, several equations of the characteristics of individual vehicle queues were also developed. These equations can be used to estimate the maximum and average queue lengths of a vehicle queue for a given time period, the time needed to clear individual vehicles from a vehicle queue, and the total and average traffic delays of a vehicle queue.
INTRODUCTION

A work zone is defined in the Highway Capacity Manual (1) as “an area of highway in which maintenance and construction operations are taking place that impinge on the number of lanes available to moving traffic or affect the operational characteristics of traffic flowing through the area.” It is common knowledge that each year a highway construction season starts with establishing work zones on roadways and that the work zones cause traffic delays. In order to efficiently plan and schedule work zone operations, it is essential to accurately estimate traffic delays at work zones. Traffic delays at work zones reflect the work zone impact on traffic flows and are the basis for calculating excess user costs. They can be estimated by various methods, such as simulation and mathematical equations.

This paper presents the results of a study on traffic delays at work zones on Indiana freeways. The derivations of equations for calculating various components of work zone traffic delays are described and the applications of these equations are illustrated through an example using traffic data collected from an Indiana freeway work zone. Traffic delays at a work zone include delays caused by deceleration of vehicles while approaching the work zone, reduced vehicle speed through the work zone, time needed for vehicles to resume freeway speed after exiting the work zone, and vehicle queues at the work zone. Vehicle queues occur when traffic flow is higher than the traffic capacity of the work zone. Because of the randomness of traffic flow, vehicle queues may also form even when traffic flow is below the work zone capacity. The traffic delay equations were developed in this study for both when the arrival traffic flow is above the work zone capacity and below it. It should be pointed out that traffic delays at work zones could be caused by many other factors, such as weather conditions and incidents, which
are not discussed in this paper. In addition to equations of work zone traffic delays, several equations of the characteristics of individual vehicle queues were also developed. These equations can be used to estimate the maximum and average queue lengths of a vehicle queue for a given time period, the time needed to clear individual vehicle in a vehicle queue, and the total and average traffic delays of a vehicle queue.

**TRAFFIC CAPACITY, FLOW AND SPEED AT WORK ZONES**

The values of traffic capacity, vehicle speed and flow rate at work zones are essential for estimating traffic delays. These traffic measures were previously determined (2) for work zones in Indiana on four-lane divided highways. The types of work zones are shown in Figures 1 and 2 and defined as follows (3):

1. Partial Closure (or single lane closure) - when one lane in one direction is closed, resulting in little or no disruption to traffic in the opposite direction.

2. Crossover (or two-lane two-way traffic operations) - when one roadway is closed and the traffic which normally uses that roadway is crossed over the median, and two-way traffic is maintained on the other roadway.

A partial closure work zone disrupts traffic in only one direction and a crossover work zone affects traffic in both directions (the median crossover direction and the opposite direction). However, a crossover work zone allows the construction crew to work on two lanes and also provides a safer work area because the work area is separated from traffic. At a partial closure work zone, the closed lane can be either the left lane or the right lane, depending on the need of construction work. At a crossover work zone, traffic flows in both the opposite direction and the
median crossover direction are affected by the work zone. Therefore, the desired traffic measures of work zones were obtained for four types of work zone layouts, including partial closure with the right lane closed, partial closure with the left lane closed, crossover in the opposite direction, and crossover in the crossover direction.

Appropriate values of work zone capacity, traffic flow rate and speed must be used to accurately estimate traffic delays at work zones. The mean values of these traffic measures, as shown in Table 1, were obtained and discussed in detail in a separate paper (2). These values can be used as typical values in estimating traffic delays at work zones if actual values are not available for individual work zones. It was observed that traffic flows at Indiana freeway work zones changed from uncongested to congested conditions always with a sharp speed drop. Therefore, work zone capacity is defined in this study as “the traffic flow rate just before a sharp speed drop followed by a sustained period of low vehicle speed and fluctuating traffic flow rate”.

To express work zone capacity in passenger car per hour, the traffic flow rate was converted to hourly volume and the adjustment factors from the 1994 Highway Capacity Manual were used to convert trucks and buses to passenger car equivalents. It was observed at the work zones on Indiana freeways that, under uncongested traffic conditions, vehicle speed remains relatively stable with minor fluctuations and vehicles pass through the work zone smoothly. When traffic is congested at a work zone, the mean vehicle speed remains lower than uncongested speed and the mean traffic flow rate is below the work zone capacity. The traffic flow rate at a work zone during congestion is called the queue-discharge rate of the work zone because it is actually the rate of the queued vehicles being discharged from the work zone.
To show the variations of the observed traffic flow rates and vehicle speeds at the selected work zones, the work zone traffic data is summarized in Table 2 with values of maximum, minimum, mean, and standard deviation.

**TRAFFIC DELAY ESTIMATION**

Traffic delays at work zones are caused by reduced number of lanes for traffic and lower vehicle speed. Traffic delays consist of those under uncongested traffic condition and those under congested traffic condition. When the traffic volume exceeds the work zone capacity, traffic congestion occurs and, therefore, results in vehicle queues and traffic delays. On the other hand, when the traffic volume is below the work zone capacity, vehicles may pass a work zone smoothly but at a lower speed than the normal freeway speed. Vehicles at this reduced speed need a longer time to pass the work zone than to pass the same length of the roadway without a work zone. This additional time spent at the work zone is also a traffic delay caused by the work zone. Furthermore, because of the stochastic feature of traffic flow, traffic queues may also form at a work zone even when the traffic flow rate is below the work zone capacity. All these types of traffic delays at work zones should be accounted for and estimated to examine the impact of work zones on highway traffic and the resulting user costs.

**Delay Due To Vehicle Deceleration Before Entering Work Zone**

Assuming a uniform deceleration, the delay for each vehicle before entering a work zone can be calculated using the basic equations of dynamics. Although vehicle deceleration before a work zone is not exactly uniform in reality, this assumption was made to simplify the derivation
of the corresponding delay equation. Since vehicles decelerate gradually, this assumption is believed to be reasonable and should result in a fairly accurate estimation of the vehicle delay time. Without a work zone, the travel time \( t_f \) of a vehicle over a section of length \( s \) at the freeway speed \( v_f \) is:

\[
t_f = \frac{s}{v_f}
\]  

(1)

With a work zone, the approach travel time \( t_a \) of the vehicle with a uniform deceleration over the same section to reduce its speed from the freeway speed \( v_f \) to the work zone speed \( v_z \) is:

\[
t_a = \frac{2s}{v_f + v_z}
\]  

(2)

Then the delay \( d_d \) due to deceleration (from \( v_f \) to \( v_z \)) of a vehicle when approaching a work zone is:

\[
d_d = t_a - t_f = \frac{2s}{v_f + v_z} - \frac{s}{v_f}
\]  

(3)

This delay is called deceleration delay because it occurs when vehicles decelerate before entering a work zone. The average distance over which the vehicles decelerate from the freeway speed to the work zone speed was measured as 1.4 kilometers using a Global Positioning System (GPS) at Indiana freeway work zones. This average value should be used in Equation 3 to calculate the delay.
Delay Due To Reduced Speed Through Work Zone

The traffic delay when vehicles travel through a work zone is the difference between the travel time needed to pass the work zone at the reduced speed and the travel time to pass the same length of the roadway without a work zone at the normal freeway speed. If the length of a work zone is L, then the delay \( d_z \) of a vehicle travelling within the work zone can be calculated as:

\[
d_z = L \left( \frac{1}{v_z} - \frac{1}{v_f} \right)
\]  

(4)

Delay For Resuming Freeway Speed After Exiting Work Zone

Vehicles travel at the reduced speed through a work zone and accelerate to their original freeway speed after exiting the work zone. The extra time for this speed resuming is a delay compared to freeway traffic without a work zone interruption. If the average acceleration is denoted as \( a \), then the distance (S) traveled to change speed from \( v_z \) to \( v_f \) is:

\[
S = \frac{v_f^2 - v_z^2}{2a}
\]  

(5)

The time needed for a vehicle to accelerate from \( v_z \) to \( v_f \) is

\[
t_1 = \frac{v_f - v_z}{a}
\]  

(6)

If there is no work zone, the time needed for a vehicle to travel the same distance is
\[ t_2 = \frac{S}{v_f} = \frac{v_f^2 - v_z^2}{2a v_f} \]  \hspace{1cm} (7)

Therefore, the delay for a vehicle to accelerate to its original speed is the difference between \( t_1 \) and \( t_2 \):

\[ d_a = t_1 - t_2 = \frac{v_f - v_z}{a} - \frac{v_f^2 - v_z^2}{2a v_f} = \frac{(v_f - v_z)^2}{2a v_f} \]  \hspace{1cm} (8)

In a separate study, the acceleration rates at Indiana freeway work zones were measured through a GPS device. The average values of the acceleration rates for different work zone types are calculated and presented in Table 3.

**Delay Due To Vehicle Queues**

1. *Vehicle Queues during Uncongested Traffic*

   Vehicle queues at a work zone can occasionally form even when the traffic volume is less than the work zone capacity. This type of delay is attributed to the stochastic nature or the randomness of traffic flow. It can be analyzed and estimated using queuing theory (4).

   Queuing theory is used to predict the characteristics of a queuing system mathematically. A queuing system consists of a servicing facility, a process of arrival of customers to be served by the facility, and the process of service. For a queuing system, it is necessary to specify the following system characteristics and parameters:

   1. Input process -- average rate of arrival and statistical distribution of time between arrivals;
2. Service mechanism -- service time average rates and distribution and number of customers that can be served simultaneously;

3. Queue discipline -- to the rules followed by the server in taking the customers into service, such as “first-come, first-served”, or “random selection for service”.

A notational representation is often used to describe the input distribution, service time distribution, and the number of servers of a queuing system. It is written as: Input distribution/Service time distribution/Number of servers. Some standard notations used in queuing theory include G for an arbitrary distribution, M for Poisson (if arrivals) or exponential distribution (for interarrival or service times), D for a constant length of time (for interarrival or service times). For example, M/M/1 represents a queuing system with Poisson arrivals, exponentially distributed service times, and one server.

To estimate traffic delays with queuing theory, a work zone can be modeled as a server for vehicles to enter the work zone in order of the vehicle arrivals. A work zone with one lane open is thus a one server queuing system and the queue discipline is apparently first-come first-served. The average arrival rate of the vehicles is the traffic flow rate and the service rate of the system is the traffic capacity of the work zone. Because of the randomness of highway traffic, the queuing system can be represented as a system with Poisson arrivals, exponentially distributed service times, and one server. That is, a freeway work zone with one lane open can be modeled as a M/M/1 queuing system.

If the average arrival rate of vehicles is denoted as $F_a$, then the average interval between arrivals is $1/F_a$. If the service rate of the system is the work zone capacity $F_c$, the average service time is $1/F_c$. The ratio $\rho = F_a/F_c$ is called the traffic intensity. If $\rho < 1$ (that is, $F_a < F_c$,...
or the traffic flow rate is below the work zone capacity), the vehicle queues can be mathematically estimated with queuing theory. On the other hand, if \( \rho \geq 1 \) (traffic flow rate exceeds the work zone capacity), queuing theory cannot be used to analyze queues.

In this queuing system, the vehicles in the queuing system are defined as those vehicles that have already merged from the closed lane into the open lane leading to the work zone. Based on queuing theory (4 and 5), the average number of vehicles in the system is

\[
E(n) = \frac{\rho}{1 - \rho} = \frac{\frac{F_a}{F_c}}{F_c - F_a} \quad \text{for } \rho < 1 \tag{9}
\]

The average waiting time that an arrival vehicle spends before entering the work zone is

\[
d_w = E(w) = \frac{F_a}{F_c (F_c - F_a)} \tag{10}
\]

The average queue length (or the average number of vehicles in the waiting line) is

\[
E(m) = \frac{F_a^2}{F_c (F_c - F_a)} \tag{11}
\]

Equation 11 is the average queue length over all time, including the period when there is no queue (i.e., queue length is 0). In practice, it is more helpful to know the average vehicle queue length if there is indeed a vehicle waiting line before the work zone. This is defined as the average queue length, given that the queue length is greater than 0. The equation for estimating this queue length is

\[
\bar{Q} = E(m \mid m > 0) = \frac{F_c}{F_c - F_a} \tag{12}
\]

In analyzing traffic delays at work zones, Equations 10 and 12 can be utilized to estimate the average vehicle delay time and the average queue length under uncongested traffic conditions.
2. Vehicle Queues during Congested Traffic

Traffic congestion occurs when the traffic flow rate exceeds the work zone capacity. As given in Table 2, under congested traffic conditions, the average speeds are lower than under uncongested traffic conditions and the average flow rates are below the capacity values. Apparently, these average values of speeds and flow rates should be used in estimating work zone traffic delays under congested traffic conditions.

Once the flow rate of arrival vehicles exceeded the work zone capacity, for a given time period the number of vehicles arriving would be larger than the number of departing at the work zone. The difference between the number of vehicles arrived and the number of vehicles departed is the vehicle queue formed at the work zone. This can be written as

\[ Q = (F_a - F_d) t \]  \hspace{1cm} (13)

where

- \( t \) = time;
- \( Q \) = vehicle queue formed during time \( t \);
- \( F_a \) = traffic flow rate of arrival vehicles;
- \( F_d \) = vehicle queue-discharge rate (traffic flow rate of departure vehicles during congestion).

If there was an original queue (\( Q_0 \)) at the beginning of the time (\( t=0 \)), then the total queue length at time \( t \) is

\[ Q_t = Q_0 + (F_a - F_d) t \]  \hspace{1cm} (14)
If vehicles arrive at a constant rate and depart the work zone at the vehicle queue-discharge rate within a given hour, then the total vehicle queue at the end of hour $i$ can be calculated as follows:

$$Q_i = Q_{i-1} + F_{ai} - F_d$$  \hspace{1cm} (15)

where $Q_i =$ total vehicle queue at the end of hour $i$;

$F_{ai} =$ hourly volume of arrival vehicles at hour $i$;

$F_d =$ vehicle queue-discharge rate.

This equation is equivalent to

$$Q_m = Q_0 + \sum_{i=1}^{m} (F_{ai} - F_d) = Q_0 + \sum_{i=1}^{m} F_{ai} - mF_d$$  \hspace{1cm} (16)

It should be pointed out that, in Equation 15 or 16, $F_{ai}$ and $F_d$ are hourly traffic volumes under congested traffic conditions and time $t$ is not explicitly expressed because it equals 1.0 hour. If time $t$ is less than 1.0, i.e., $t$ is somewhere between hour $i-1$ and hour $i$, the equation should be written as

$$Q_i(t) = Q_{i-1} + (F_{ai} - F_d) t$$  \hspace{1cm} (17)

$Q_i(t)$ represents the vehicle queue length at time $t$ within hour $i$, where $t$ is measured starting at the beginning of hour $i$.

In Equation 15, only if $F_{ai} > F_d$, the queue length increases during hour $i$, or $Q_i > Q_{i-1}$. On the other hand, if $F_{ai} < F_d$, the queue length decreases during hour $i$, or $Q_i < Q_{i-1}$. If the calculated $Q_i$ from Equation 15 is less than 0, it implies that $F_{ai}$ is less than $F_d$ and that the vehicle queue
has dissipated at some point in time within hour i. If $Q_i = 0$ from Equation 15, then the queue dissipated exactly at the end of hour i. If $Q_i < 0$, then the queue was cleared at a time point $t$ before the end of hour i. Setting Equation 17 equal to 0, i.e., $Q_i(t) = Q_{i-1} + (F_{ai} - F_d) t = 0$, the time $t$ at which the last vehicle in the queue was cleared can be obtained as

$$t = \frac{Q_{i-1}}{F_d - F_{ai}}$$

(18)

Here $t$ is less than 1.0 hour because the queue was cleared before the end of hour i.

The traffic delay associated with the queued vehicles can be calculated based on the vehicle queue lengths. As given in Equation 17, the vehicle queue length at time $t$ within hour i is $Q_i(t) = Q_{i-1} + (F_{ai} - F_d) t$. The delay (in vehicle-hours) of these $Q_i(t)$ vehicles during an infinitesimal time interval $[t, t + \Delta t]$ within hour i can be expressed as

$$\Delta D_i = Q_i(t) \Delta t$$

(19)

The total traffic delay during a time period from $t = 0$ to $t = T$ is

$$D_i^{t+T} = \int_0^T \Delta D_i dt = \int_0^T [Q_{i-1} + (F_{ai} - F_d) t] dt = Q_{i-1}T + \frac{1}{2}(F_{ai} - F_d)T^2$$

(20)

For $T = 1$, then Equation 20 results in the total delay (in vehicle-hours) in hour i, that is

$$D_i = Q_{i-1} + \frac{1}{2}(F_{ai} - F_d)$$

(21)

If the traffic congestion started at hour 1 and ended during hour I, then $D_1, D_2 \ldots D_{I-1}$ can be calculated with Equation 21. Because the traffic congestion ended during hour I, the time $t$, at which the last vehicle in the queue was cleared, should be first calculated using Equation 18.
With $t_i$, the delay can be estimated using Equation 20.

$$D_{i=1} = D_1 = Q_{i-1} \left( \frac{Q_{i-1}}{F_d - F_{a_d}} \right) + \frac{1}{2} \left( F_{a_d} - F_d \right) \left( \frac{Q_{i-1}}{F_d - F_{a_d}} \right)^2$$

or

$$D_i = \frac{Q_{i-1}^2}{2(F_d - F_{a_d})}$$

In the Texas study, Memmott and Dudek (6) obtained results similar to Equations 21 and 23 using a graphic method. However, a significant difference is that their study used work zone capacity, instead of queue-discharge rate, as the flow rate of departure vehicles. As can be seen in Table 1, the values of work zone capacities are higher than those of queue-discharge rates. It was observed at the Indiana freeway work zones that traffic flow rates could not sustain the capacity level during traffic congestion. Therefore, using the capacity values as the departure traffic flow rates would result in under estimations of the work zone traffic delays.

**Total Traffic Delay At Work Zone**

The total traffic delay at a work zone is then the sum of the individual delays discussed above. Under uncongested traffic conditions, the total traffic delay at a work zone in hour $i$ is

$$DELAY_i = F_{ai} \left( d_d + d_z + d_a + d_u \right)$$

or

$$DELAY_i = F_{ai} \left( d_d + d_z + d_a \right) + D_i$$

Under congested traffic conditions, the total delay at a work zone in hour $i$ is

$$DELAY_i = F_{ai} \left( d_d + d_z + d_a \right) + D_i$$
As Equation 22 shows, traffic congestion exists only during a portion \((t_f)\) of the last hour (hour I). Therefore, the total delay in hour I should include the discharged queued vehicles during the first portion of the hour \((t_f)\) and the expected vehicle queues due to the randomness of traffic flow during the second portion of the hour \((1-t_f)\).

\[
\text{DELAY}_I = F_{ai} [d_{d} + d_{e} + (1-t_f)d_{e}] + D_I
\]  

(26)

where \(t_f\) and \(D_I\) are defined in Equations 22 and 23, respectively.

**EQUATIONS OF VEHICLE QUEUE CHARACTERISTICS**

In addition to traffic delay estimations, also derived are the equations of other characteristics of vehicle queues caused by traffic congestion. These equations can be utilized to calculate such values as maximum and average queue lengths, time needed to clear a given vehicle queue, and waiting time of vehicles in queue.

According to Equation 15, \(Q_i\) increases as long as \(F_{ai}\) is greater than \(F_d\). Therefore, the maximum queue length occurs just before \(F_{ai}\) drops below \(F_d\). For example, if \(F_{ai} > F_d\) during hour 0 through hour I-1 and \(F_{ai} < F_d\) at hour I, then the maximum of the vehicle queue up to hour I is \(\max(Q) = Q_{I-1}\).

At the beginning of hour \(i\) the queue length (in number of vehicles) is \(Q_{i-1}\), and at the end of hour \(i\) the queue length is \(Q_i\). According to Equation 17, the queue length changes linearly with time within each hour. Therefore, the average queue length of hour \(i\) is the mean of \(Q_{i-1}\) and \(Q_i\):
\[
\bar{Q}_i = \frac{1}{2}(Q_{i-1} + Q_i) = Q_{i-1} + \frac{1}{2}(F_{ai} - F_d)
\]  \hspace{1cm} (27)

It is interesting to note that Equation 27 is the same as Equation 21, however, they have different dimensions. \( \bar{Q}_i \) is queue length in number of vehicles and \( D_i \) is traffic delay in vehicle-hours.

Queue length at any time \( t \) between hour \( i-1 \) and hour \( i \) is given by Equation 17 as \( Q_i(t) \), which is the number of vehicles in the waiting line (or queue). Therefore, when a vehicle arrived at time \( t \), this vehicle became the \( Q_i(t) \) th vehicle in the queue. That is, the queue length at time \( t \) is \( Q_i(t) = Q_{i-1} + (F_{ai} - F_d) t \). Since the queue-discharge rate is \( F_d \) and the number of vehicles in the queue at time \( t \) is \( Q_i(t) \), the time needed to clear all \( Q_i(t) \) vehicles from the queue is

\[
W_i = \frac{Q_i(t)}{F_d} = \frac{Q_{i-1} + (F_{ai} - F_d) t}{F_d}
\]  \hspace{1cm} (28)

\( W_i \) is also the waiting time for the \( Q_i(t) \) th vehicle to be cleared from the queue. This waiting time is nothing but the delay incurred to the vehicle that arrived at time \( t \). Therefore, Equation 28 can be used to estimate the delay for any vehicle after it joined the queue. The values of \( W_i \) are not only important to traffic engineers, but also important to motorists. For example, the values of \( W_i \) can be displayed on a variable message sign as the “expected delay time” at the work zone.

Because delay for the \( n \)th vehicle is given as \( \frac{n}{F_d} \) by Equation 28, the total delay of all \( Q_i(t) \) vehicles in the queue is
\[ W_{\text{total}} = \frac{1}{F_d} + \frac{2}{F_d} + \cdots + \frac{Q_i(t) - 1}{F_d} + \frac{Q_i(t)}{F_d} = \frac{Q_i(t)[1 + Q_i(t)]}{2F_d} \]  

(29)

or

\[ W_{\text{total}} = \frac{[Q_{i-1} + (F_{ai} - F_d)t] \times [1 + Q_{i-1} + (F_{ai} - F_d)t]}{2F_d} \]  

(30)

It should be emphasized that \( W_{\text{total}} \) obtained from Equation 29 or Equation 30 is the total delay counted from time \( t \), because the vehicles that joined the queue before time \( t \) had already sustained delays between the time they arrived and time \( t \). The average delay time per vehicle in the queue (counted from time \( t \)) is then equal to the total queue delay, \( W_{\text{total}} \), divided by the total number of vehicles in the queue, \( Q_i(t) \).

\[ W_{\text{avg}} = \frac{W_{\text{total}}}{Q_i(t)} = \frac{1 + Q_i(t)}{2F_d} \]  

(31)

While Equation 28 provides the expected delay time for a given queue length, Equations 29, 30, and 31 are useful for answering other frequently asked questions by highway engineers related to vehicle queue lengths. For example, a highway engineer would ask “what is the estimated total delay (or average delay) for a queue of 100 vehicles?” without giving detailed information on arriving times of individual vehicles in the queue. Thus, Equation 29, or 30, or 31 can be used to calculate the estimated delays of the vehicle queue. This is practical because the arriving times of individual vehicles in a queue are difficult to collect and are usually not available to highway engineers. If, however, the arriving time for each of the \( n \) vehicles in the queue is known as \( t_j \), where \( j \) denotes the \( j \)th vehicle in the queue \((j = 1, 2, 3, \ldots, n)\), then the queue length at time \( t_j \) is \( Q(t_j) \) and the total delay of the \( n \) vehicles can be calculated as.
\[ W_{total} = \frac{Q(t_1)}{F_d} + \frac{Q(t_2)}{F_d} + \ldots + \frac{Q(t_{j-1})}{F_d} + \frac{Q(t_j)}{F_d} + \frac{Q(t_{n-1})}{F_d} + \frac{Q(t_n)}{F_d} = \frac{\sum_{j=1}^{n} Q(t_j)}{F_d} \] (32)

Similar to Equation 31, the average delay per vehicle in the queue is derived as

\[ W_{avg} = \frac{W_{total}}{Q(t_n)} = \frac{\sum_{j=1}^{n} Q(t_j)}{F_d \times Q(t_n)} \] (33)

As can be seen, Equations 32 and 33 provide the total and average delays since the first vehicle in the queue arrived, while Equations 29 through 31 give the delays since the last vehicle in the queue arrived. To use Equations 32 and 33, the arriving times of all vehicles in the queue must be recorded.

AN APPLICATION EXAMPLE OF THE TRAFFIC DELAY EQUATIONS

To demonstrate the applications of the derived traffic delay equations, these equations were applied to calculate the traffic delays at a freeway work zone in Indiana during a 24-hour period. The work zone was a crossover work zone of 11.7 kilometers long on Interstate 70 (I-70) between State Road 9 and State Road 29. As a crossover work zone (Figure 2) affects traffic in both directions, the traffic delays at the work zone were calculated for both the median crossover and the opposite directions. In calculating the traffic delays, the values of work zone capacities and queue-discharge rates listed in Table 1 and the observed average vehicle speeds were used. It should be pointed out that if the actual vehicle speeds were not available, the values of the mean vehicle speeds in Table 2 could be used as the default values in the traffic delay calculations. However, because of the variations in vehicle speeds through work zones, it is
recommended to use the observed vehicle speed values in the calculations of work zone traffic delays whenever they are available in order to improve the quality of the traffic delay estimation.

The hourly arrival traffic data and the calculated traffic delays are presented in Tables 4 and 5. The adjustment factors from the 1994 Highway Capacity Manual were used to convert trucks and buses to passenger car equivalents. Therefore, the traffic flow rates are expressed in passenger cars per hour. The traffic delays listed in the two tables are the hourly delays of the individual and total delays. It was calculated that the total 24-hour delay was 2922 car-hours in the crossover direction and 2380 car-hours in the opposite direction. The average hourly delay was 122 car-hours and 99 car-hours in the two directions, respectively. Thus, the whole work zone (including both directions) caused a total traffic delay of 5302 car-hours over the 24 hours. The average hourly traffic delay at the work zone was 221 car-hours.

To illustrate the changes of traffic delays with time, the traffic delays over the 24-hour period are plotted in Figures 3 and 4. Figure 3 indicates that in the crossover direction the traffic delays caused by reduced speed and vehicle queues were the two major sources of the total traffic delay. These two traffic delays increased considerably during the traffic congestion hours when vehicle queue formed and vehicle speed decreased. In the opposite direction of the work zone, as shown in Figure 4, the reduced speed delay was significantly greater than other delays. Figure 4 also shows that the reduced speed delay increased during the hours when the traffic flow rate was high and the vehicle speed was low. The queue delay was not significant in the opposite direction because the traffic flow rates did not exceed the work zone capacity during the 24 hours.

The equations of vehicle queue characteristics were also utilized to calculate additional queue attributes during traffic congestion in the crossover direction. The maximum queue
length occurred at 17:00 with a length of 304 equivalent passenger cars. Since the queue length at 16:00 was 293 equivalent passenger cars, the average queue length during 16:00-17:00 was calculated with Equation 27 as

$$\bar{Q}_i = \frac{1}{2}(Q_{i-1} + Q_i) = \frac{1}{2}(293 + 304) = 299\ \text{equivalent passenger cars.}$$

Using Equation 28, the time needed to clear all 304 queued vehicles was calculated as

$$W_t = \frac{Q_i(t)}{F_d} = \frac{304}{1587} = 0.19\ \text{hours} = 11.5\ \text{minutes.}$$

Using Equation 29, the total delay of these queued vehicles (counted from time 17:00) was calculated as

$$W_{total} = \frac{Q_i(t)[1 + Q_i(t)]}{2F_d} = \frac{304 \times (1 + 304)}{2 \times 1587} = 29\ \text{car-hours.}$$

The average delay time per vehicle in the queue (counted from time 17:00) can be calculated by Equation 31:

$$W_{avg} = \frac{W_{total}}{Q_i(t)} = \frac{1 + Q_i(t)}{2F_d} = 0.096\ \text{hours} = 5.8\ \text{minutes.}$$

**CONCLUSIONS**

This paper presented the derivations and applications of a series of equations of work zone traffic delays and vehicle queue attributes. It has been emphasized that in calculating traffic delays the vehicle queue-discharge rates, instead of the work zone capacity, should be used because the queue-discharge rates are lower than the work zone capacity. The application of the derived equations in the I-70 work zone indicated that the reduced speed delay was a
major contributor to the total traffic delay for both directions of the work zone. It should be noted that the I-70 work zone was a relatively long one (11.7 kilometers). Because the reduced speed delay is directly proportional to the work zone length, a shorter work zone would result in a considerably less speed delay. The sample problem showed that the vehicle queue delay was a major component of the total delay. However, vehicle queues under uncongested traffic condition, or the stochastic queues, caused much less traffic delays than vehicle queues during traffic congestion.

The equations of the characteristics of individual vehicle queues can be used to estimate the maximum and average queue lengths of a vehicle queue for a given time period, the time needed to clear individual vehicles from a vehicle queue, and the total and average traffic delays of a vehicle queue. The quantities of these vehicle queue attributes have a number of useful applications. For example, one such application could display real-time traffic information on the variable message signs to inform motorists of expected delay through the work zone. The values of expected queue lengths and delay time can also be applied for adaptive traffic controls at work zones.

Although only the traffic delay issues are discussed in this paper, it should be emphasized that the safety of both motorists and highway workers should always be the first priority in a work zone. Therefore, the safety issues in a work zone must be considered before taking any actions to reduce the work zone traffic delay.
ACKNOWLEDGMENTS

This study was supported by the Indiana Department of Transportation and the Federal Highway Administration through the State Planning and Research Program.
REFERENCES


6. Memmott, J. L., and Dudek, C. L. *QUEWZ – 85, A Model to Calculate the Road User Costs at Work Zone*.” Texas Transportation Institute, Texas A&M University, College Station, Texas, 1982.
### TABLE 1 Mean Values of Work Zone Capacities, Queue-Discharge Rates and Vehicle Speeds

<table>
<thead>
<tr>
<th>Work Zone Type</th>
<th>Mean Capacity (passenger cars/hour)</th>
<th>Mean Queue-Discharge Rate (passenger cars/hour)</th>
<th>Mean Speed During Uncongestion</th>
<th>Mean Speed During Congestion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crossover (Opposite Direction)</td>
<td>1745</td>
<td>1393</td>
<td>90 km/hour</td>
<td>40 km/hour</td>
</tr>
<tr>
<td>Crossover (Crossover Direction)</td>
<td>1612</td>
<td>1587</td>
<td>92 km/hour</td>
<td>40 km/hour</td>
</tr>
<tr>
<td>Partial Closure (Right Lane Closed)</td>
<td>1537</td>
<td>1216</td>
<td>95 km/hour</td>
<td>50 km/hour</td>
</tr>
<tr>
<td>Partial Closure (Left Lane Closed)</td>
<td>1521</td>
<td>1374</td>
<td>92 km/hour</td>
<td>63 km/hour</td>
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# TABLE 2  Summarized Traffic Flow and Speed Data at Work Zones

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<th>Work Zone Type</th>
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<th>Minimum Observed Value</th>
<th>Maximum Observed Value</th>
<th>Mean</th>
<th>Standard Deviation</th>
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### TABLE 3 Mean Values of Vehicle Acceleration Rate

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<th>Mean Acceleration Rate (km/h/s)</th>
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<td>Merging</td>
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<tr>
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<td>Non-Merging</td>
<td>1.131</td>
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**Note:**
Merging -- Test vehicle performed merging maneuver from the closed lane to the open lane when approaching to work zone.
Non-Merging – Test vehicle did not perform merging maneuver (approached to work zone from the open lane).
### TABLE 4  Estimated Traffic Delays at the I-70 Work Zone (in the Crossover Direction)

<table>
<thead>
<tr>
<th>Time</th>
<th>Speed (km/h)</th>
<th>Traffic Flow (Passenger Cars/Hour)</th>
<th>Deceleration Delay (Car-Hours)</th>
<th>Reduced Speed Delay (Car-Hours)</th>
<th>Acceleration Delay (Car-Hours)</th>
<th>Queue Delay (Car-Hours)</th>
<th>Total Delay (Car-Hours)</th>
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<tr>
<td>Time</td>
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<td>Traffic Flow (Passenger Cars/Hour)</td>
<td>Deceleration Delay (Car-Hours)</td>
<td>Reduced Speed Delay (Car-Hours)</td>
<td>Acceleration Delay (Car-Hours)</td>
<td>Queue Delay (Car-Hours)</td>
<td>Total Delay (Car-Hours)</td>
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FIGURE 1 Partial Closure Work Zone
FIGURE 2  Crossover Work Zone
Figure 3. Traffic Delays at the I-70 Work Zone (Crossover Direction)
Figure 4. Traffic Delays at the I-70 Work Zone (Opposite Direction)